Name of College - S.S. College J-Bad
Deptt - Mathematics
Topic :→ Problem based on Direction Cosine (contd) (Analytical Geometry of 3-D)
Class - B.Sc I (Sub + H)
Date - 30-07-2020 Time - 1:00 P.M to 1:45 P.M

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* **Problem** :→ Show that st. lines whose direction cosines are given by the Equations

$$l + m + n = 0$$

and $2mn + 3nl - 5lm = 0$ are at right angles

Solution :→ By the question

Here $l + m + n = 0$

$$\Rightarrow l = -(m+n) \quad \text{——— (1)}$$

Also $2mn + 3nl - 5lm = 0$

$\Rightarrow 2mn - 3n(m+n) + 5(m+n)m = 0$

putting $l = -(m+n)$

$\Rightarrow 2mn - 3mn - 3n^2 + 5m^2 + 5mn = 0$

$\Rightarrow 5m^2 + 4mn - 3n^2 = 0$

it is a quadratic equation in m.

$$\therefore m = \frac{-4n \pm \sqrt{16n^2 + 60n^2}}{10} = \frac{-4 \pm 2\sqrt{19}}{10}n$$

$$= \frac{-2 \pm \sqrt{19}}{5}n$$

then $m_1 = \frac{-2 \pm \sqrt{19}}{5} n_1$ $m_2 = -\frac{2-\sqrt{19}}{5} n_2$

When $m_1 = \frac{-2+\sqrt{19}}{5} n_1$, $m_2 = \frac{-2-\sqrt{19}}{5} n_2$

from (1) $l_1 = \frac{-3+\sqrt{19}}{5} n_1$ $l_2 = \frac{-3-\sqrt{19}}{5} n_2$

If θ be the angle between them

then $\theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$

$= \cos^{-1}\left\{\frac{9-19}{25} n_1 n_2 + \frac{4-19}{25} n_1 n_2 + n_1 n_2\right\}$

$\cos^{-1}\left\{-\frac{10}{25} n_1 n_2 - \frac{15}{25} n_1 n_2 + n_1 n_2\right\}$

$= \cos^{-1}\left[-\left[\frac{10}{25} + \frac{15}{25} - 1\right] n_1 n_2\right]$

$= \cos^{-1} 0 = \pi/2.$

Problem - Find the angle between two lines whose direction co-sines are given by the equations

$l+m+n=0$; $l^2+m^2-n^2=0$

Solution $\rightarrow$ The solution of the two equations gives two values of l, m, n, let (l_1, m_1, n_1) and (l_2, m_2, n_2) are the values of l, m, n obtained by the equation

$l+m+n=0$ and $l^2+m^2-n^2=0$

and then angle between the two lines

$= \cos^{-1}[l_1 l_2 + m_1 m_2 + n_1 n_2]$

On Elimination n between the two Equations

$l^2 + m^2 - (l+m)^2 = 0$

$\Rightarrow 2lm = 0$

Either $l = 0$ or $m = 0$

if $l = 0 \Rightarrow m + n = 0 \therefore m = -n$

But $l^2 + m^2 + n^2 = 1$

$0 + m^2 + m^2 = 1$

$m^2 = \frac{1}{2}$

$m = \pm \frac{1}{\sqrt{2}}$

the Direction Co-sines of One line is

$(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

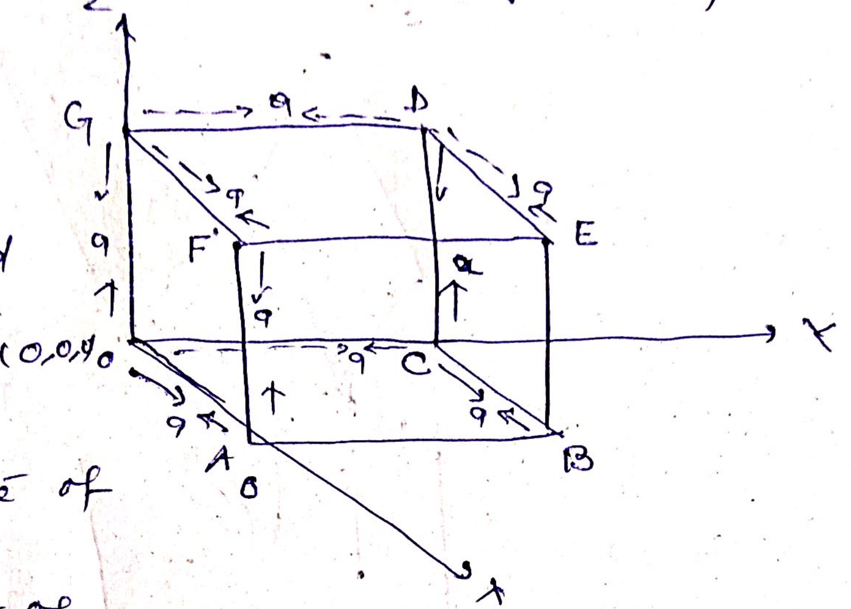
if $m = 0$, the Direction Cosines of the other line are $(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}})$

$\therefore \theta = \cos^{-1}\left[0 + 0 \pm \frac{1}{2}\right]$

$= \cos^{-1}(\pm \frac{1}{2})$

$\therefore \theta = \pi/3$ or $2\pi/3$

Find the angle between two diagonals of a cube :→



Let each side of cube be a.

Let us take O as the origin and OA, OC, OG as the x, y, z axes respectively.

The co-ordinate of O $= (0,0,0)$

The co-ordinate of E $= (a, a, a)$

∴ Direction ratio of OE $= (a, a, a)$

and direction cosines of OE $= \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}$

$= \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

The co-ordinate of A $= (a, 0, 0)$

and co-ordinate of D $= (0, a, a)$

Direction ratio of AD $= (-a, +a, +a)$

Direction cosines of AD $= \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

If θ be the angle between the diagonal OE and AD

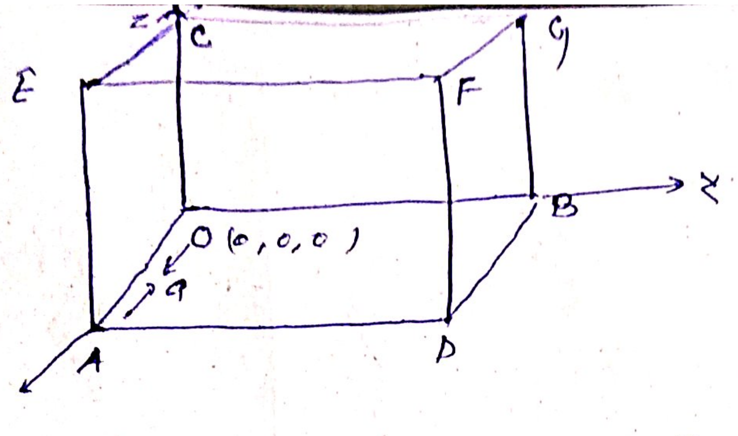
using $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$\cos\theta = -\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1}{3}$

$\therefore \theta = \cos^{-1}\frac{1}{3}$

Problem :→ If the edges of a rectangular parallelopiped are a, b, c, show that the angle between the four diagonals are given by

$$\cos^{-1}\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}$$



Let us take one of the vertices O of the Parallelopiped as the origin and the three planes as three Rectangular faces.

Let $OA = a$, $OB = b$ $OC = c$

$\therefore$ Co-ordinate of A $= (a, 0 0)$

" " B $= (0, b, 0)$

" C $= (0, 0, c)$

" D $= (a, b, 0)$

" E $= (a, 0, c)$

" G $= (0, b, c)$

" F $= (a, b, c)$

Direction cosines of diagonal OF are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Direction cosines of AG

$$= \frac{-a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Direction cosines of CD $= \frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{-c}{\sqrt{a^2+b^2+c^2}}$

Direction cosines of BE $= \frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{-b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

Angle between OF and AG is $\cos\theta =$ using $l_1 l_2 + m_1 m_2 + n_1 n_2$

$\cos^{-1} \frac{-a^2+b^2+c^2}{a^2+b^2+c^2}$